ECE 457 Final Project Analog Portion
An Analysis of the Impact of Frequency and Phase Offset in the Demodulation of QAM and SSB Signals

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***Abstract*—This project simulates both Quadrature Amplitude Modulation (QAM) and Single-Sideband Modulation (SSB) communication methodologies through Matlab. The process starts with modulating and demodulating a signal using each method and showing the perfect reconstruction of the transmitted AC signal. This perfect reconstruction occurs with no frequency or phase offset, or noise, in the system. Next, we perform the same modulations, but including first a frequency offset and then a phase offset. We observe the interference, or cross talk, that occurs between two signals when these offsets are present. We analyse each method for their robustness to these respective offsets, and then perform each modulation method on a pair of audio files of human speech, as well as a pair of audio files of modern music.**

I. Introduction

In 1915 John R. Carson theorized and later patented SSB modulation through a purely simply mathematical derivation of the method [1]. Early on, many thought that the idea of sidebands were merely a mathematical fiction, but overtime, this mathematical derivation was proven very much not fictional. Since 1918, SSB transmission has been a standard in carrier-telephone development throughout the world [1].

SSB modulation is not without limitation, however. If the transmitted signal has significant spectral content near DC, SSB is difficult to generate. For applications, such as transmitting speech waveforms, where there is little relevance to the DC components of a signal, this does not matter very much and SSB modulation is sufficient. For systems where near DC components are highly important, such as transmitting and receiving a picture where spectral content near DC indicates a dark color, SSB modulation is not adequate, however [2]. This phenomena led to the adoption of QAM for certain communication systems.

Another benefit of QAM is that it can be exactly generated without the need for sharp-cutoff bandpass filters [2]. The synchronizations between transmitter and receiver becomes more important, however, which we will see in this paper. Improper synchronization will lead to cochannel interference of two signals, which makes QAM less robust than SSB modulation. The benefit of QAM is that it does not have a phase discontinuity at DC and two signals can be communicated with the simplest system, compared to only one signal for SSB modulation.

The spectral efficiency of SSB modulation and QAM are identical, so the decision of which method to use between the two is usually a matter of what signals are being transmitted and an individual systems requirements. This paper will look into some of the similarities and differences of each communication method.

II. System Description

A. SSB Modulation and Demodulation

We will first discuss SSB modulation. Our SSB modulation and demodulation that we will discuss here will focus on a lower sideband (LSB) derivation.

We start with our message signal $m(t)$. We generate the double-sideband signal (DSB) with a desired carrier frequency by multiplying $m(t)$ with $cos(2\pi f_c t)$. The carrier frequency must be large enough so that there is no interference between the lower and upper ends of the DSB signal.

$$S_{DSB}(t) = m(t) \cdot cos(2\pi f_c t) \tag{1}$$

Since we are performing SSB modulation, we are only interested in either the upper sideband (USB) or LSB of the DSB signal. Here, we will perform SSB with the LSB signal by filtering out the USB with a bandpass filter.

$$S_{LSB}(t) = BPF(S_{LSB}(t)) \tag{2}$$

After filtering, our signal is modulated and is ready to be sent through our communication channel through an antenna or other transmitter.

The signal is then received by another antenna or receiver and needs to be demodulated so that the original signal can be reconstructed.

To reconstruct our original signal, we take our received signal and multiply it by $2cos(2\pi fct)$. This shifts our signal in the frequency domain back to center and multiplying it by two returns the amplitude of the signal back to what it previously was. We then filter this signal with a low pass filter to eliminate the part of the signal that is shifted to an even higher frequency after multiplying by $2cos(2\pi f_c t)$.

$$\hat{m(t)} = LPF(S_{LSB}(t) \cdot 2cos(2\pi f_c t)) \tag{3}$$

Where, when there is no frequency or phase shift, $\hat{m(t)} = m(t)$.

For discussions sake, we will explore the effects of frequency and phase offset in our demodulation. This can be represented by the following demodulation equation.

$$\hat{m(t)} = LPF(S_{LSB}(t) \cdot 2cos(2\pi(f_c + \Delta f)t + \theta)) \tag{4}$$

In this paper, we will look at the effects of Δf and θ independently. Obviously, when Δf and θ are non zero, $\hat{m(t)}$ is not necessarily equal to $m(t)$. This concludes the system description for SSB modulation.

B. QAM Modulation and Demodulation

As mentioned in the Introduction section, QAM allows for modulating, sending, and demodulating two signals at a time. Therefore, for this discussion, we will consider two message signals, $m_1(t)$ and $m_2(t)$.

In order to generate the QAM signal, we multiply $m_1(t)$ by the in-phase carrier, $cos(2\pi f_c t)$, and $m_2(t)$ by the quadrature carrier, $sin(2\pi f_c t)$. We then sum these two signals, represented by $S_{QAM}(t)$. This is in part what makes QAM unique; since sine and cosine are perpendicular to each other, if there is no frequency or phase offset anywhere in the system, the summation of the two signals will have no interference with each other.

$$S_{QAM}(t) = m_1(t) \cdot cos(2\pi f_c t) + m_2(t) \cdot sin(2\pi f_c t) \tag{5}$$

The summed signal is passed over the channel, received, and demodulated.

The demodulation process occurs by multiplying $S_{QAM}(t)$ with the local oscillator $2cos(2\pi f_c t)$ and then filtering out the higher frequency components to obtain $m_1(t)$, and similarly, multiplying $S_{QAM}(t)$ with the local oscillator $2sin(2\pi f_c t)$ and then filtering out the higher frequency components to obtain $m_2(t)$

$$\hat{m_1(t)} = LPF(S_{QAM}(t) \cdot 2cos(2\pi f_c t)) \tag{6}$$

$$\hat{m_2(t)} = LPF(S_{QAM}(t) \cdot 2sin(2\pi f_c t)) \tag{7}$$

Where, when there is no frequency or phase shift, $\hat{m_1(t)} = m_1(t)$, and $\hat{m_2(t)} = m_2(t)$, since there is perfect reconstruction.

Similarly, as stated in the above section, for discussions sake, we will explore the effects of frequency and phase offset in our demodulation. This can be represented by the following demodulation equations.

$$\hat{m_1(t)} = LPF(S_{QAM}(t) \cdot 2cos(2\pi(f_c + \Delta f)t + \theta)) \tag{8}$$

$$\hat{m_2(t)} = LPF(S_{QAM}(t) \cdot 2sin(2\pi(f_c + \Delta f)t + \theta)) \tag{9}$$

We will look at the effects of Δf and θ for QAM independently as well. Obviously, when Δf and θ are non zero, $\hat{m_1(t)}$ is not necessarily equal to $m_1(t)$ and $\hat{m_2(t)}$ is not necessarily equal to $m_2(t)$. This concludes the system description for SSB modulation.

The differences of $\hat{m_1(t)}$ with $m_1(t)$ and $\hat{m_2(t)}$ with $m_2(t)$ for QAM will be compared with the differences of $\hat{m(t)}$ with $m(t)$ for SSB, and the robustness of each with respect to frequency and phase offset will be analyzed.

III. Noise Effect Analysis

A. Effect of frequency and phase shift in SSB

Equation 4 gives a good understanding of the effect of frequency and phase offset in demodulation in SSB. From Equation 4, we can expand our equation to the following:

$$\begin{aligned} \hat{m(t)} = LPF(&m(t)cos(2\pi(f_c + \Delta f)t + \theta) \\ &-m_h(t)sin(2\pi(f_c + \Delta f)t + \theta)) \end{aligned} \tag{10}$$

Where $m_h(t)$ is the Hilbert transform of $m(t)$.

We first analyse phase offset. Looking at Equation 10, we can see that the phase offset will weaken the strength of the signal until a phase of 90° is achieved, in which the signals magnitude is near 0. Since cosine is periodic, as the phase approaches 180° from 90°, the magnitude increases again, and this increasing and decreasing in magnitude occurs periodically as the phase offset varies between multiples of 0 through 2π.

The frequency offset in SSB is slightly more difficult to conceptualize. Δf will shift the frequency range up so that the entire signal is not captured in its original form, but shifted away form DC in the frequency domain. The lowpass filter may then filter out some of these higher frequency components. This leads to some higher frequencies being dropped, and the signal itself being distorted, tending in general to higher frequencies, but not linearly. Maybe the most important feature is that Δf will alter the periodicity of the cosine component of SSB demodulation, leading to an oscillatory decrease in the magnitude of the signal, with a frequency of this oscillatory decrease dictated by Δf, where the frequency of "throbbing" of the magnitude of the signal increases with Δf.

B. Effect of frequency and phase shift in QAM

Equation 8 and Equation 9 give a good understanding of the effect of frequency and phase offset in demodulation in QAM. We will look at phase offset of QAM first.

Starting with Equation 8 and Equation 9, since we are only concerned with phase, we set Δf to 0, and applying trig identities, we can find that:

$$\hat{m_1(t)} = LPF(m_1(t) \cdot cos(\theta) - m_2(t) \cdot sin(\theta)) \quad (11)$$

$$\hat{m_2(t)} = LPF(m_2(t) \cdot cos(\theta) - m_1(t) \cdot sin(\theta)) \quad (12)$$

It is rather straight forward to see from the above equations that crosstalk will occur between $m_1(t)$ and $m_2(t)$. For $m_1(t)$, as θ increases to $90°$, the magnitude of $m_1(t)$ will decrease and $m_2(t)$ will increase. When θ is $90°$, $\hat{m_1(t)}$ is actually going to be equivalent to $m_2(t)$, since $-sin(90°) = 1$. This same relationship is observed for $m_2(t)$, except obviously with the signals reversed.

Again, analyzing the effect of Δf is slightly more complicated. From Equation 8 and Equation 9, we let θ equal 0, since we are only concerned about the effect of Δf. Applying trig identities, we can find that:

$$\hat{m_1(t)} = LPF(m_1(t) \cdot cos(2\pi\Delta ft) - m_2(t) \cdot sin(2\pi\Delta ft)) \quad (13)$$

$$\hat{m_2(t)} = LPF(m_2(t) \cdot cos(2\pi\Delta ft) - m_1(t) \cdot sin(2\pi\Delta ft)) \quad (14)$$

Following a similar explanation to the SSB Δf, we can see how the Δf leads to a "throbbing" of the magnitude of the desired signal, and also introduces a "throbbing" effect of crosstalk from the non-desired signal. This is because the magnitude of the cosine and sine components of each oscillate with time, coming in and out of phase with a rate based on Δf; the frequency of said "throbbing" increases as Δf increases.

We now understand the effects of frequency and phase shifts in both SSB modulation and QAM. This now means that we can look at the simulation results of our simulated SSB modulations and QAM and understand the corresponding plots.

IV. Simulation Results

A. SSB Simulation Results

We will first look at some simulations of SSB modulation and demodulation. Here, our $m(t) = 2cos(2\pi(100)t) + 3cos(2\ pi(1000)t)$.

As we can see in Fig. 1 and Fig. 2, with no offset of phase or frequency, we have perfect reconstruction of $m(t)$.

Fig. 3 shows the effect of phase offset on the demodulation. We can see that the magnitude is lessened as the phase increases. Fig. 4 shows what happens at 90°phase shift, which is that the signal is lost completely.

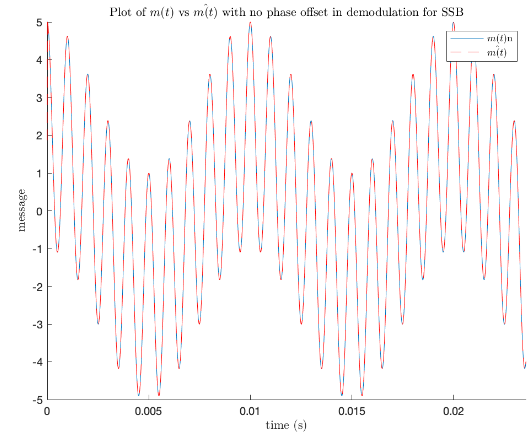

Fig. 1. SSB modulation of $m(t)$ with no phase offset

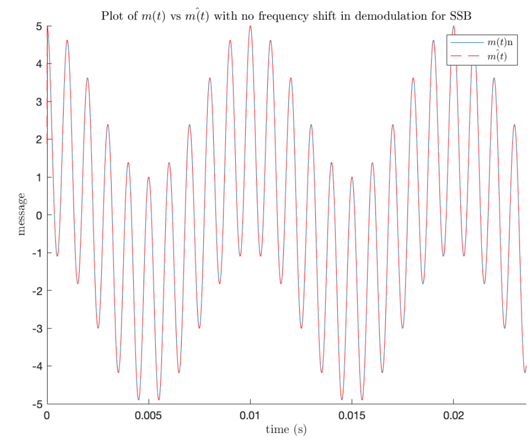

Fig. 2. SSB modulation of $m(t)$ with no frequency offset

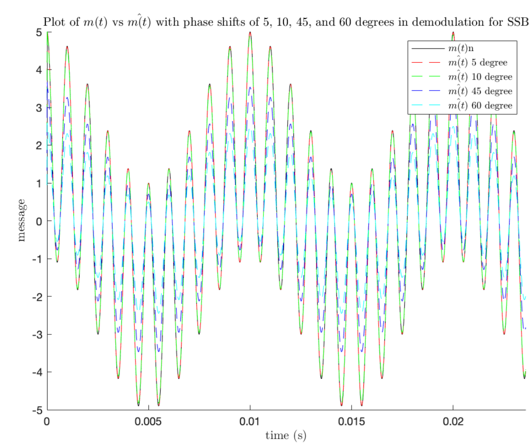

Fig. 3. SSB modulation of $m(t)$ with phase offset of 5°, 10°, 45°, and 60°

Fig. 5 and Fig. 6 show the effect of frequency offset on $m(t)$. From these two figures, we can get a picture for the "throbbing" that occurs in the signal upon demodulation, and

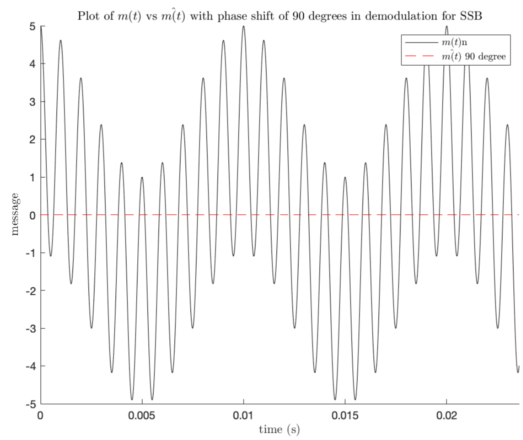

Fig. 4. SSB modulation of $m(t)$ with phase offset of 90°

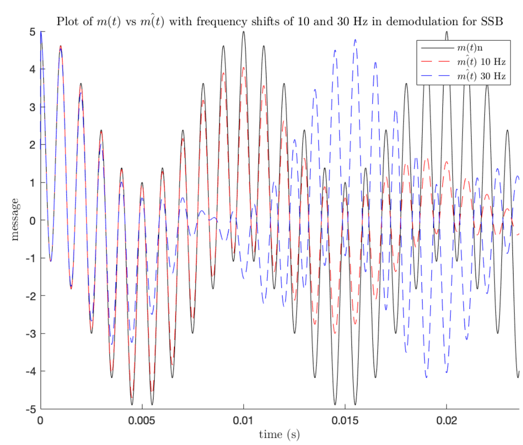

Fig. 5. SSB modulation of $m(t)$ with frequency offset of 10 Hz and 30 Hz

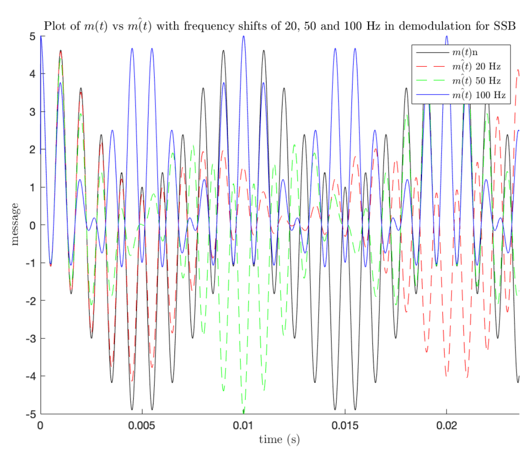

Fig. 6. SSB modulation of $m(t)$ with frequency offset of 20 Hz, 50 Hz, and 100 Hz

how the "throbbing" varies with Δf.

B. QAM Simulation Results

Next will look at some simulations of QAM modulation and demodulation. Here, our $m_1(t) = 2cos(2\pi(100)t)$ and $m_2(t) = 3cos(2\pi(1000)t)$.

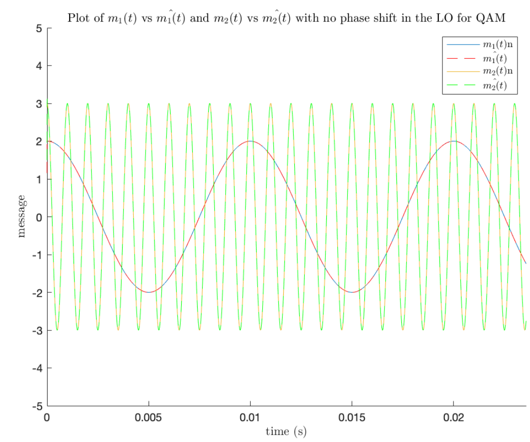

Fig. 7. QAM modulation of $m_1(t)$ and $m_2(t)$ with no phase offset

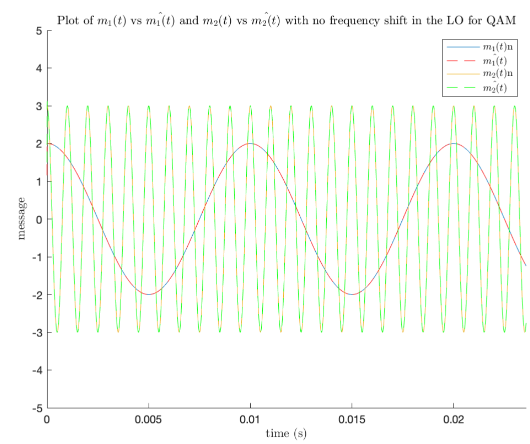

Fig. 8. QAM modulation of $m_1(t)$ and $m_2(t)$ with no frequency offset

Again, we see that with no frequency or phase shift, there is perfect capturing of the original signals, as seen in Fig. 7 and Fig. 8.

Fig. 9 and Fig. 10 shows the phase offset of QAM modulation for $m_1(t)$ and $m_2(t)$. It is clear that as the offset increases, $m_1(t)$ becomes more and more like $m_2(t)$, and likewise, $m_2(t)$ becomes more and more like $m_1(t)$.

When the phase is 90°, the demodulated $m_1(t)$ is exactly equal to $m_2(t)$, and the demodulated $m_2(t)$ is exactly equal to $m_1(t)$, as shown in Fig. 11.

Shifting our focus over to frequency offset, we look at Fig. 12, Fig. 13, Fig. 14, and Fig. 15 and see how frequency offset affects demodulation. As seen in these figures, we can observe the the signal basically coming in and out of phase due to the Δf, as explained in a previous section.

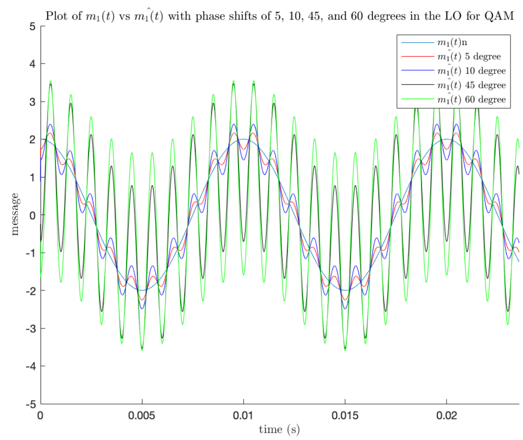

Fig. 9. QAM modulation of $m_1(t)$ with phase offset of 5°, 10°, 45°, 60°

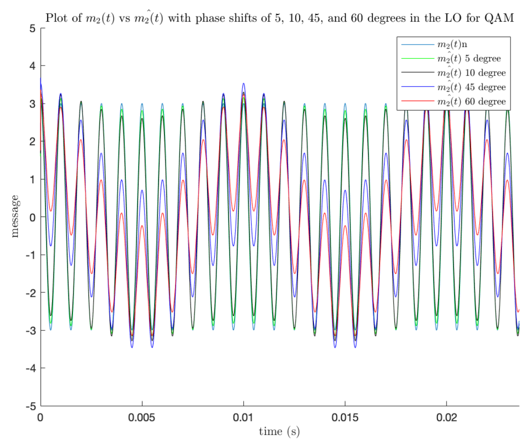

Fig. 10. QAM modulation of $m_2(t)$ with phase offset of 5°, 10°, 45°, 60°

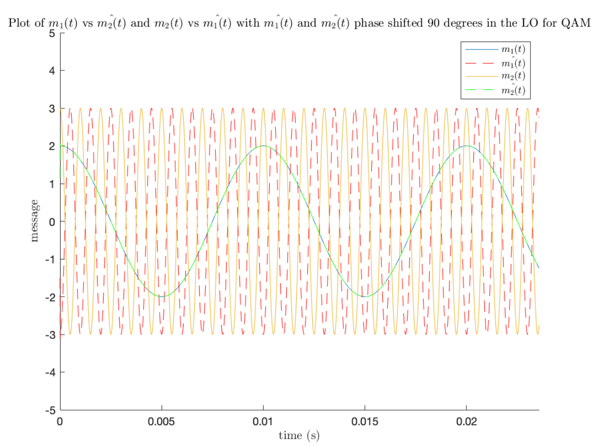

Fig. 11. QAM modulation of $m_1(t)$ and $m_2(t)$ with phase offset of 90°

Fig. 12 shows this coming in and out of phase particularly well, and is a good visual to understand the "throbbing" of the signal, explained above.

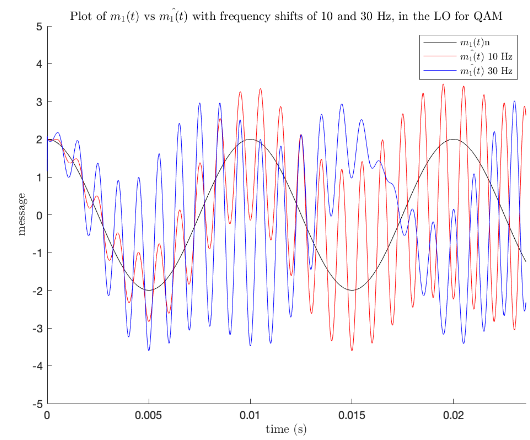

Fig. 12. QAM modulation of $m_1(t)$ with frequency offset of 10 Hz and 30 Hz

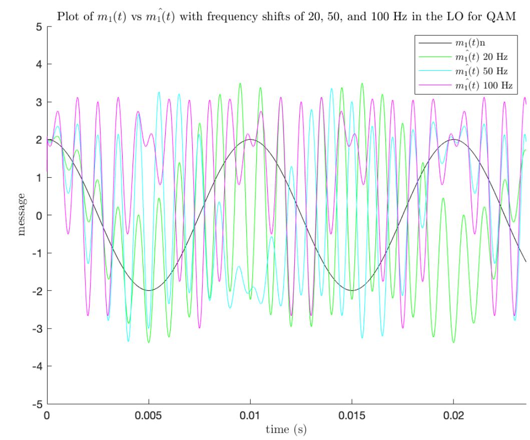

Fig. 13. QAM modulation of $m_1(t)$ with frequency offset of 20 Hz, 50 Hz, and 100 Hz

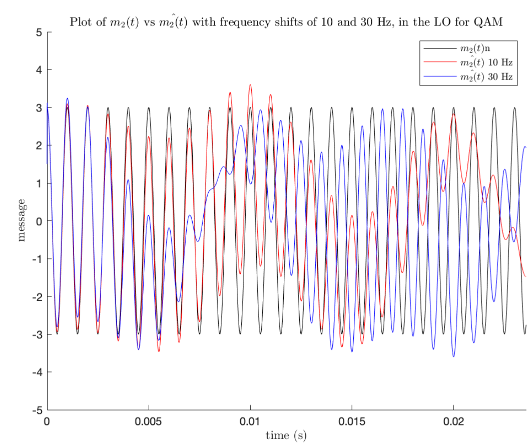

Fig. 14. QAM modulation of $m_2(t)$ with frequency offset of 10 Hz and 30 Hz

C. MSE of SSB and QAM Simulation Results

An additional thing to consider from our simulations is the Mean Squared Error (MSE) of the phase and frequency offset

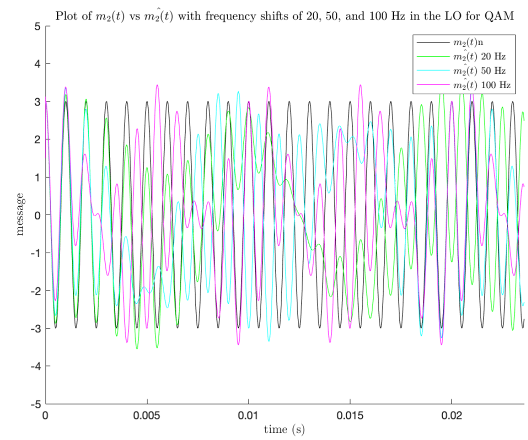

Fig. 15. QAM modulation of $m_2(t)$ with frequency offset of 20 Hz, 50 Hz, and 100 Hz

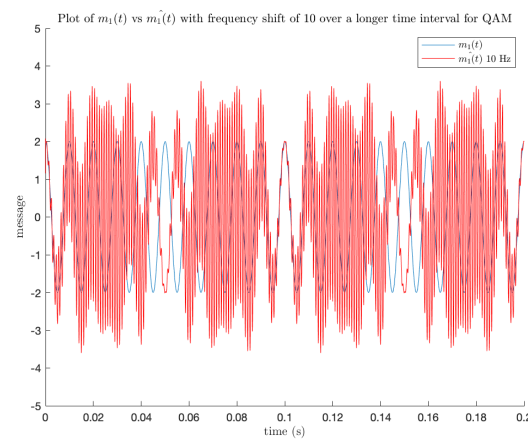

Fig. 16. QAM modulation of $m_1(t)$ with frequency offset of 10 Hz over extended interval

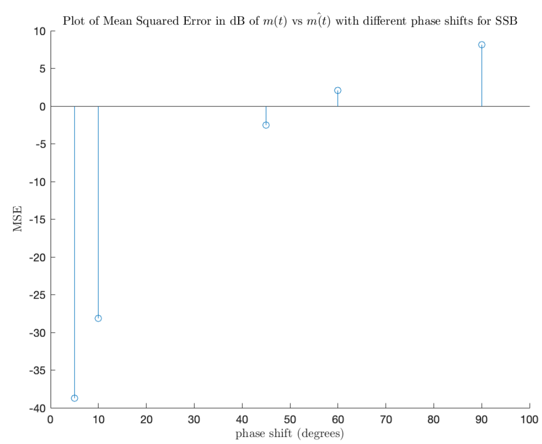

Fig. 17. SSB MSE with different phase offsets

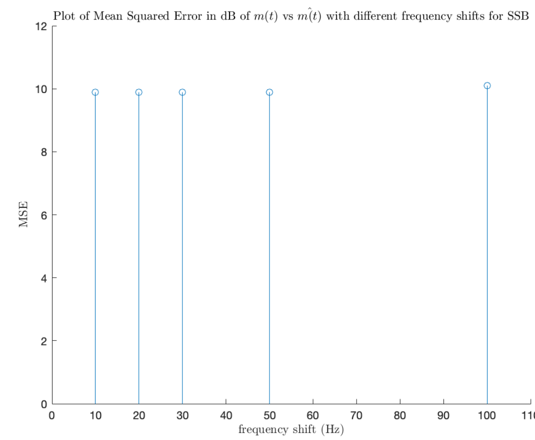

Fig. 18. SSB MSE with different frequency offsets

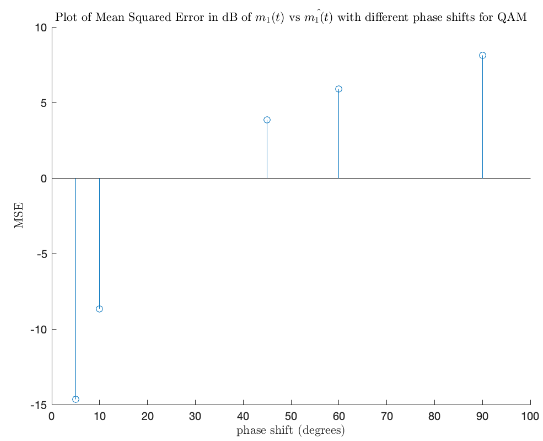

Fig. 19. QAM MSE of $m_1(t)$ with different phase offsets

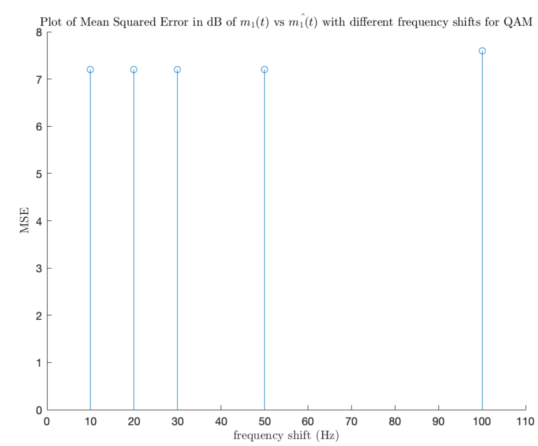

Fig. 20. SSB MSE of $m_1(t)$ with different frequency offsets

modulated signals. MSE gives us a good understanding of the error in the received and demodulated signals with phase and frequency offset. A low error indicates a robust system, and a higher error indicates a more fragile system. Looking at Fig. 17, Fig. 18, Fig. 19, Fig. 20, Fig. 21, and Fig. 20, we can start

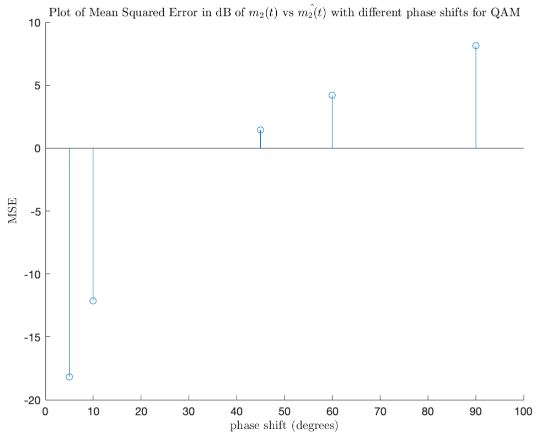

Fig. 21. QAM MSE of $m_2(t)$ with different phase offsets

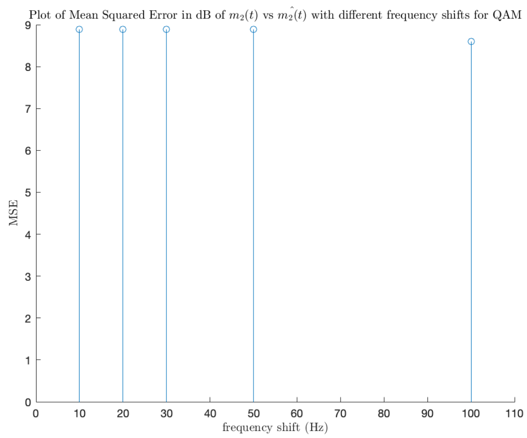

Fig. 22. SSB MSE of $m_2(t)$ with different frequency offsets

to understand each modulation schemes robustness to phase and frequency offsets.

First analysing for phase offset, we direct our attention to Fig. 17, Fig. 19, and Fig. 21. We can see that both systems are somewhat robust to a small phase offset, and that both systems also struggle more as the phase offset increases. Comparing the three figures, however, we can conclude that SSB is more robust to phase offset that QAM, as it has a lower error at low phase offsets. Additionally, we can see that both methods become more and more identical regarding MSE as the phase offset increases.

Next we analyse the MSE for different frequency offsets. Looking at Fig. 18, Fig. 20, and Fig. 22, we see that neither system does particularly well with frequency offset. Given our simulations, QAM has a slight edge in this category, but it's better performance is not significant and may change with different signals. We also see from these figures that the MSE does not increase by nearly as much with frequency offset as it did with the phase offset increasing. This means that both QAM and SSB are more robust to phase offset than frequency offset, given our simulations.

V. Extra Credit Portion

For additional exploration, we performed SSB and QAM modulation on real world audio files. First, we performed each modulation on a pair of audio files of human speech. Next, we performed each modulation on a pair of audio files of contemporary music. The two songs that were modulated were APOLOGIZE by Hykeem Jamaal Carter Jr, also known as Baby Keem [3], and DONT WANT IT, by Montero Lamar Hill, also known as Lil Nas X [4].

We start our discussion of the audio files of human speech. Similar phenomena is observed with the speech waveforms as with the original signals. The simulated images are included for reference, were we focus only on a small portion of the entire audio so that the discussed ideas can be observed. Furthermore, we try to only include figures that may provide some additional insight, as we do not want to be redundant. For formatting purposes, all of these images are included at the end of the document, following the Conclusion and References sections.

A. SSB and QAM Modulation of Human Speech Waveforms

This section pertains to Fig. 23, Fig. 24, Fig. 25, Fig. 26, Fig. 27, Fig. 28, Fig. 29, Fig. 30, and Fig. 31. We can see similar patterns to our previous simulations, but here, we can see that the discussed phenomena apply to more complex signals as well.

B. MSE of Human Speech Waveforms

This section pertains to Fig. 32, Fig. 33, Fig. 34, Fig. 35, Fig. 36, and Fig. 37. The computed MSEs follow a similar pattern to our original simulations and no discussion is necessary here.

C. MSE of Contemporary Songs

This section pertains to Fig. 38, Fig. 39, Fig. 40, Fig. 41, Fig. 42, Fig. 43, Fig. 44, and Fig. 45.

An interesting thing to note about the MSE for the songs is that the QAM actually performed better with respect to frequency offset at small offsets compared to the SSB modulation. This can perhaps be explained by each contemporary song having lots of low frequency components, and the QAM out performing SSB because the low frequencies of crosstalk were still accurate. We assume that this is due to the type of audio passed, especially since our simulations are for an idealized system. Further work may try different songs and see how they differ based on the characteristics of each.

VI. Conclusion

We have seen that both SSB modulation and QAM are effective methods for modulating and demodulating a signal for communication over a channel. Although they both have the same spectral efficiency, there are pros and cons to both. In general, we can conclude that SSB modulation is more robust to phase and frequency offset than QAM, but we know that this form of modulation is more difficult. We also know that it struggles with DC or near DC signals in real world systems. QAMs lack of robustness to phase and frequency offset is probably its greatest drawback, but the ability to communicate two signals at a time and transmit DC and near DC signals easily, makes it still a useful communication method. Ultimately, we can conclude that the decision between which of these two methods to use depends largely on what a given communication system needs. Additionally, we know that there are more complex communication protocols that can be used that might address some of the limitations of SSB modulation and QAM, but know that their improvements come at some price; the hope is that the price is one that we are happy and willing to pay, otherwise, we may be best to stick to one of the two methodologies described in this paper.

Since the formulation of SSB modulation and QAM, communications systems have improved significantly and will likely continue to improve in the years to come. Understanding where the field came from though is important to continuously move it forward, and therefore, we find the study of SSB modulation and QAM to be very useful.

References

[1] A. A. Oswald, "Early history of single-sideband transmission," *Proceedings of the IRE*, vol. 44, p. 1676–79, Dec 1956.

[2] B. P. Lathi, *Modern Digital and Analog Communication Systems*, pp. 158–167. Oxford University Press Inc., 4th ed., 2009.

[3] H. J. Carter, *APOLOGIZE*. New York City: The Orchard., 2019.

[4] M. L. Hill, *DONT WANT IT*. New York City: Columbia Records, 2021.

A. *SSB Modulation of Human Speech Waveforms*

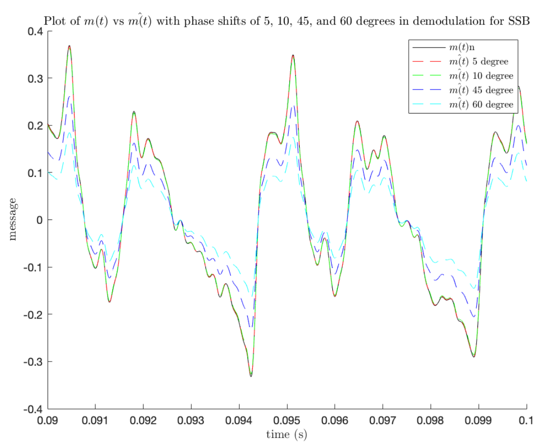

Fig. 23. SSB modulation of $m(t)$ with phase offset of 5°, 10°, 45°, and 60°for human speech

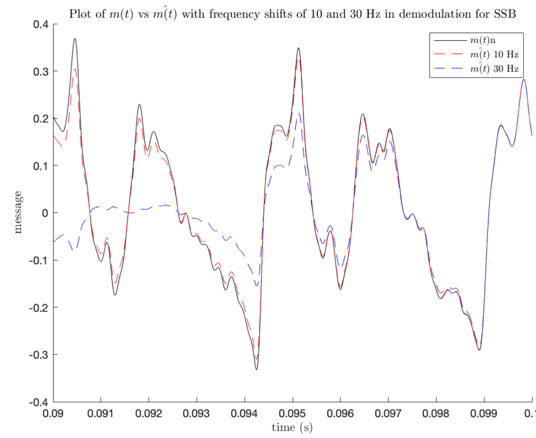

Fig. 24. SSB modulation of $m(t)$ with frequency offset of 10 Hz and 30 Hz for human speech

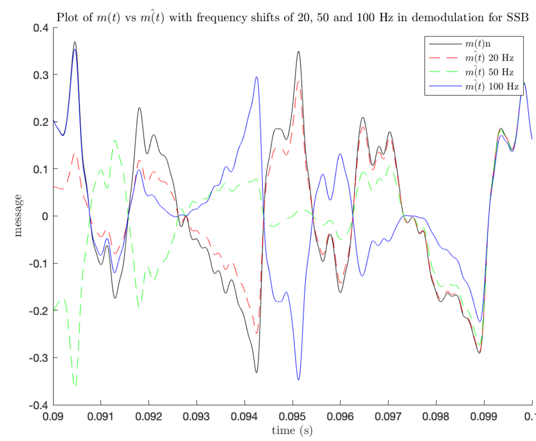

Fig. 25. SSB modulation of $m(t)$ with frequency offset of 20 Hz, 50 Hz, and and 100 Hz for human speech

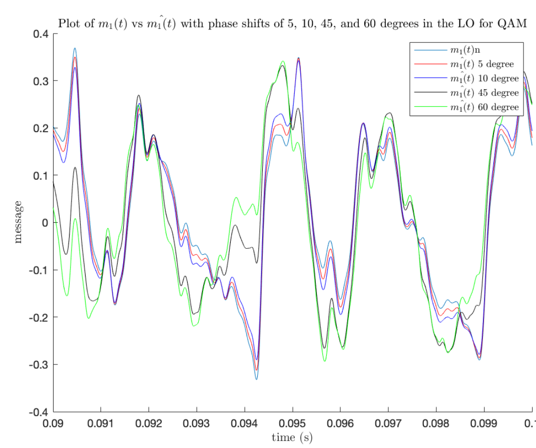

Fig. 26. QAM modulation of $m_1(t)$ with phase offset of 5°, 10°, 45°, and 60°for human speech

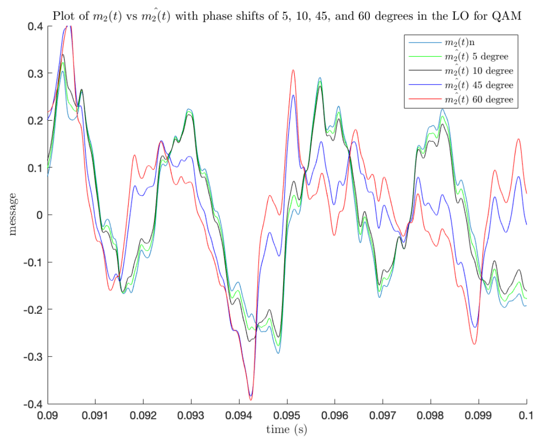

Fig. 27. QAM modulation of $m_2(t)$ with phase offset of 5°, 10°, 45°, and 60°for human speech

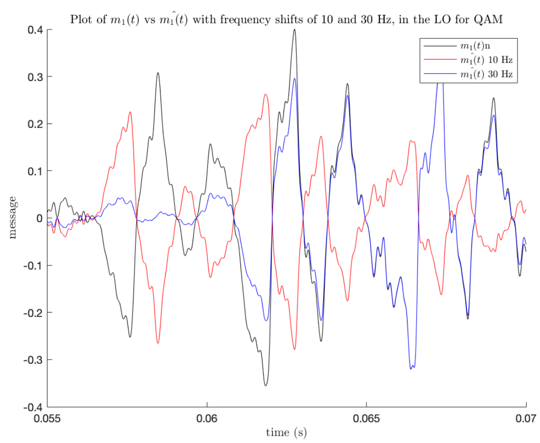

Fig. 28. QAM modulation of $m_1(t)$ with frequency offset of 10 Hz and 30 Hz for human speech

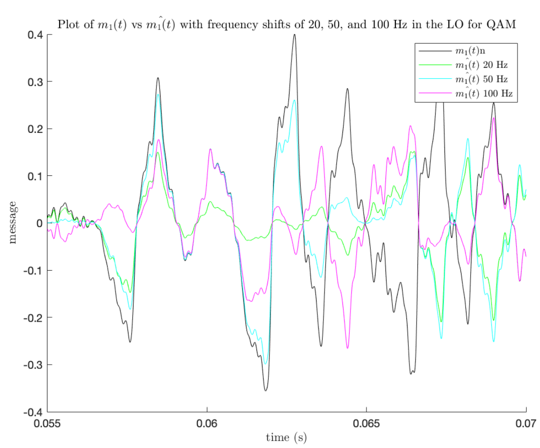

Fig. 29. QAM modulation of $m_1(t)$ with frequency offset of 20 Hz, 50 Hz, and and 100 Hz for human speech

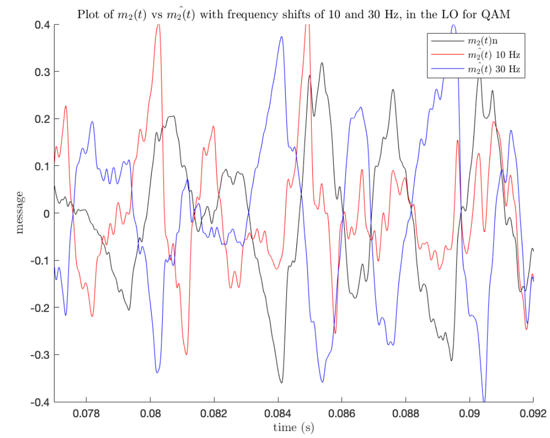

Fig. 30. QAM modulation of $m_2(t)$ with frequency offset of 10 Hz and 30 Hz for human speech

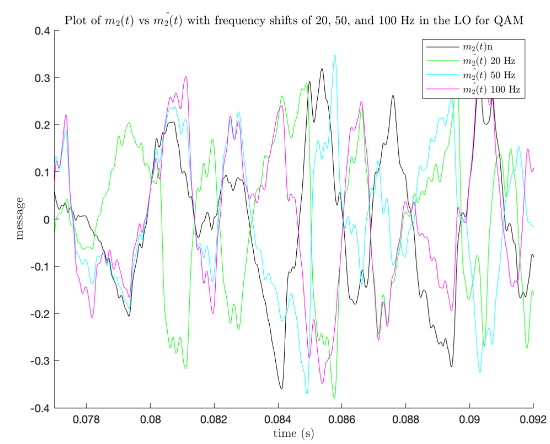

Fig. 31. QAM modulation of $m_2(t)$ with frequency offset of 20 Hz, 50 Hz, and and 100 Hz for human speech

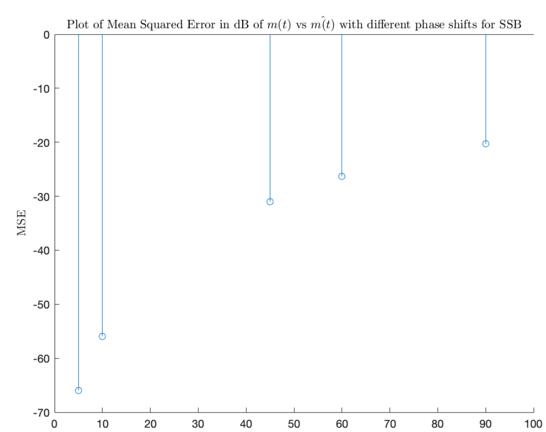

Fig. 32. SSB MSE with different phase offsets for human speech

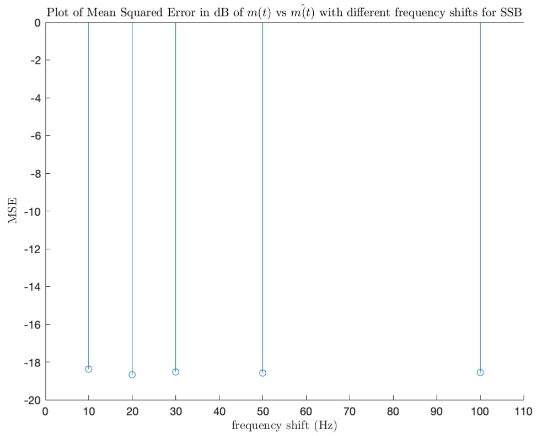

Fig. 33. SSB MSE with different frequency offsets for human speech

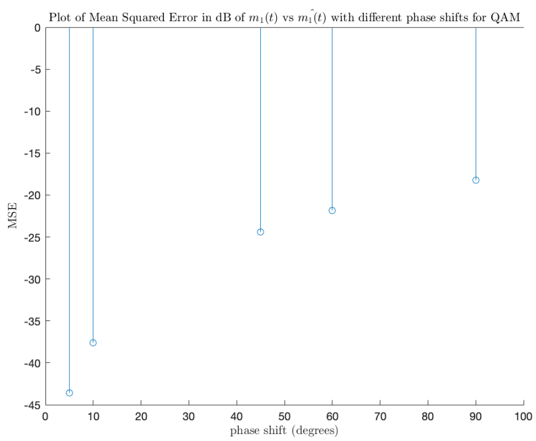

Fig. 34. SSB MSE of $m_1(t)$ with different phase offsets for human speech

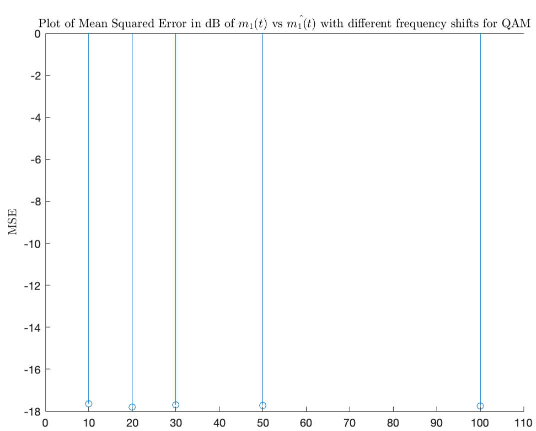

Fig. 35. QAM MSE of $m_1(t)$ with different frequency offsets for human speech

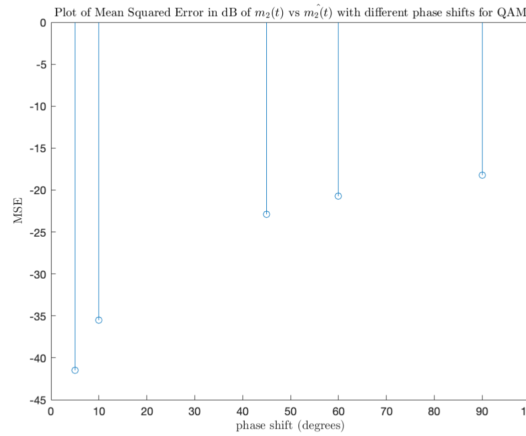

Fig. 36. SSB MSE of $m_2(t)$ with different phase offsets for human speech

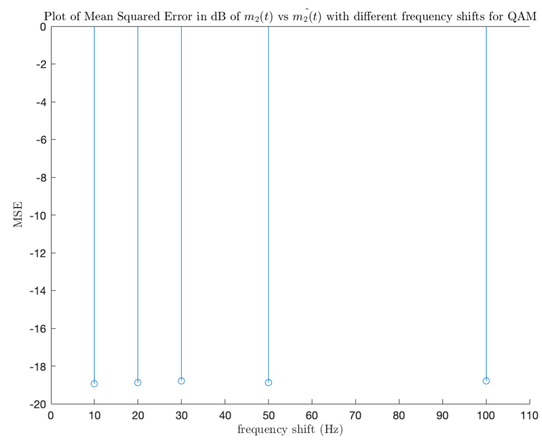

Fig. 37. QAM MSE of $m_2(t)$ with different frequency offsets for human speech

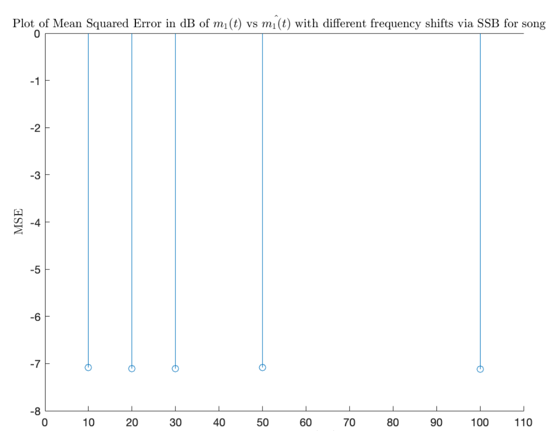

Fig. 38. SSB MSE with different phase offsets for APOLOGIZE

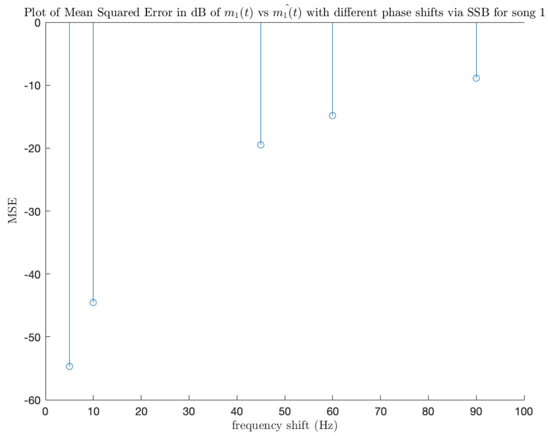

Fig. 39. SSB MSE with different frequency offsets for APOLOGIZE

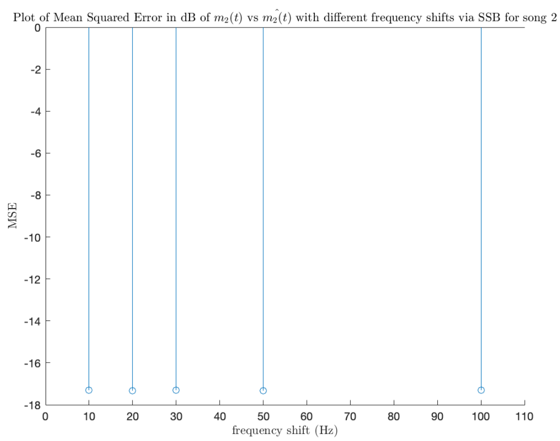

Fig. 40. SSB MSE with different phase offsets for DONT WANT IT

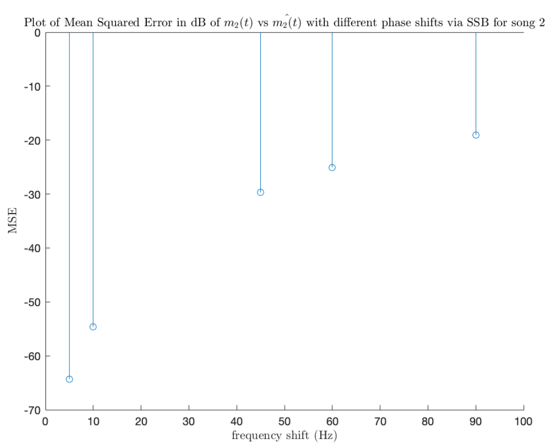

Fig. 41. SSB MSE with different frequency offsets for DONT WANT IT

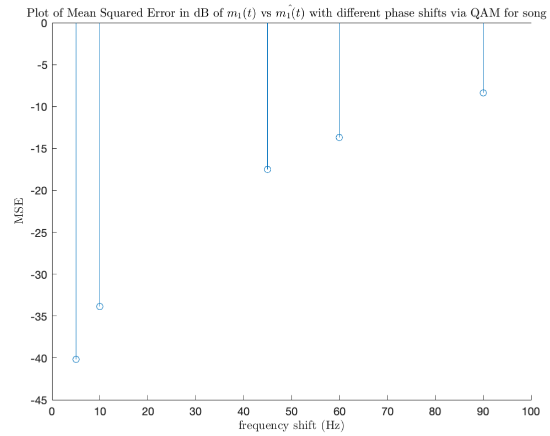

Fig. 42. QAM MSE of $m_1(t)$ with different phase offsets for APOLOGIZE

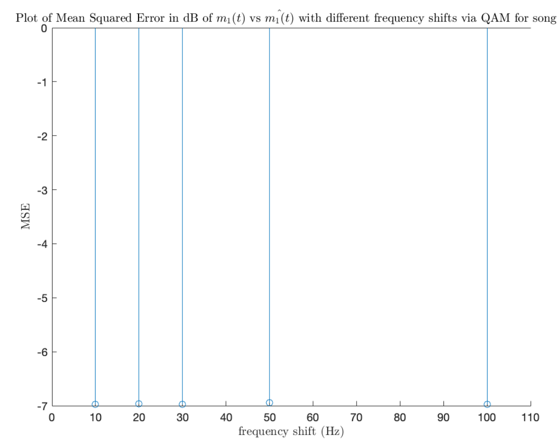

Fig. 43. QAM MSE of $m_1(t)$ with different phase offsets for APOLOGIZE

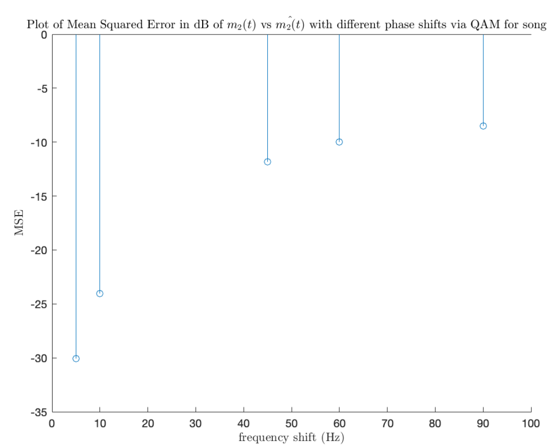

Fig. 44. SSB MSE of $m_2(t)$ with different phase offsets for DONT WANT IT

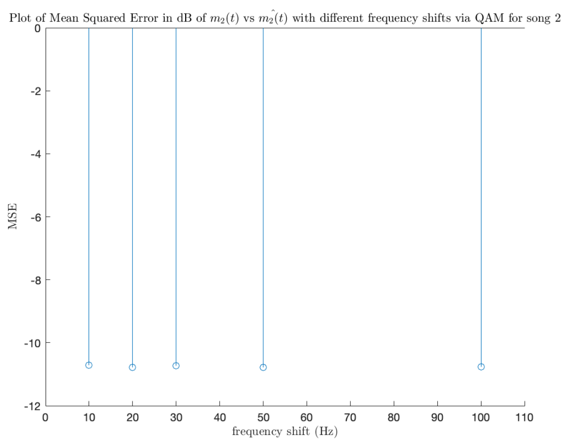

Fig. 45. QAM MSE of $m_2(t)$ with different frequency offsets for DONT WANT IT